INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH BHOPAL

Department of Chemistry

First Quiz, September 2024

Course: CHM 325/635 **Time**: 45 minutes **Full Marks** : 20

NOTE: The quiz is for a total of **20 marks**. Answer **All** questions. Marks for questions are as indicated.

1. Angular momentum $\vec{L}$ of a particle is given by $\vec{L} = m\vec{r} \times \vec{v}$. where the velocity $\vec{v}$ is related to the angular velocity $\vec{\omega}$ by $\vec{v} = \vec{\omega} \times \vec{r}$ and $\hbar, m$ are constants. Show that $\vec{L} = mr^2 \left[\vec{\omega} - \hat{r}(\hat{r} \cdot \vec{\omega}\right]$. (4 marks)

2. The expectation value of kinetic energy of a 1-particle quantum mechanical system is given by

$$\int_{all\ \ space} \psi(\vec{r}) \left(\frac{-\hbar^2}{2m} \nabla^2 \psi(\vec{r}) \right) d\tau$$

, where $\psi(\vec{r})$ is a real wavefunction that vanishes at $r \to \infty$. Show that this integral is always positive. (4 marks)

3. Let ψ be a solution of Laplace's equation over some volume V (with continuous derivatives in the volume). Prove that the integral over any closed surface in V of the normal derivative of ψ, ($\partial\psi/\partial n$, or $\vec{\nabla}\psi \cdot \vec{n}$) will be zero. (6 marks)

4. The angular momentum operator in quantum mechanics is given by $\hat{L} = -i\hbar\vec{r} \times \vec{\nabla}$. Express this operator in spherical polar coordinates. (6 marks)
