$$P = k_B T \left(\frac{\partial \ln Z}{\partial V} \right)_{T,N}$$

S for an ideal gas?

$$S = k_B \ln Z + \frac{U}{T} + \text{const.}$$

$$Z_{ideal} = \left(V \left(\frac{2\pi m}{\beta} \right)^{3/2} \right)^N$$

$$\ln Z = N \ln V + \frac{3}{2} N \ln(2\pi m) - \frac{3}{2} N \ln(\beta)$$

$$U/T = \frac{3N}{2} k_B$$

$$S = k_B \left[N \ln V + \frac{3}{2} N \ln (k_B T) + \frac{3N}{2} \right] + \text{const}$$

$$= N k_B \left[\ln V + \frac{3}{2} \ln (T) \right] + \text{const}(N)$$

$$S = k_B \left[N \ln V + \frac{3}{2} N \ln(2\pi m) - \frac{3}{2} N \ln(\beta) + \frac{3}{2} N \right] + \cancel{\text{const}} \to 0$$

$$S^{(2)}$$

$$S(2U, 2V, 2N) \stackrel{?}{=} 2\, S(U, V, N)$$

$$S^{(2)} = k_B \left[2N \ln(2V) + \frac{3}{2}(2N) \ln(2\pi m) - \frac{3}{2}(2N) \ln(\beta) + \frac{3}{2}(2N) \right] = 2 k_B \left[N \ln V + \frac{3}{2} N \ln(C) - \frac{3}{2} N \ln(\beta) + \frac{3}{2} N \right] - 2N k_B \ln 2$$

$$-2Nk_B \ln 2$$

$$Z = \sum_k e^{-\beta E_k}$$

$$P(E_k) = \frac{e^{-\beta E_k}}{Z}$$

$$\bar{E} = \sum_k P(E_k) E_k$$

$$d\bar{E} = \sum_k \left\{ dP(E_k)\, E_k + P(E_k)\, dE_k \right\}$$

$$H = -\sum_k P(E_k) \ln\left(P(E_k)\right)$$