12/8/2025 Day 5. Mathematical Structure of QM States: Vector Jpaces

utro From the 2-slit example we also note that the Es emerging at the detector were associated with a wavefunction 4 which itself resulted from the superposition of the waves 4, & 42 from the 2 stits.  $\dot{\mathcal{U}} \quad \dot{\psi}(\vec{r},t) = \dot{\psi}(\vec{r},t) + \dot{\psi}_2(\vec{r},t)$ It's easy to convince ourselves that, etrough some combination of slik & sounds, one can always represent any state of the electron in terms of superposition of other state. This superposition principle is then a feature Of PM States. Furthermore, if the wavefunction is multiplied by a Constant (say, A), then

the probability distribution implied by the state remains unchanged, Thus, the states are determined by the wavefunctions upto an arbitrary constant multiple. These observations can be bundled into our fint potulate. Postulate 1: Each state of a dynamical corresponds to a wave function, say Y. The correspondence is such that, if a state rendts from superposition 6, certain other states, the wavefunction is linearly expressible in terms of the corresponding wavefunction of the  $\Psi = c_1 \Psi_1 + c_2 \Psi_2 - \Theta$ where C,, C2 = 4 complex numbers

the same (superposed) state.  $\Psi = \Psi_{1} + \Psi_{2} = \Psi_{2} + \Psi_{1} \\
-3$   $\Psi = \Psi_{1} + (\Psi_{2} + \Psi_{3}) = (\Psi_{1} + \Psi_{2}) + \Psi_{3}$  -3We also consider, & & c & where c is an arbitrary complex number (non-rul) are corresponding to like same state. We note that there is nothing special about a probability distribution in real space. We could just as well chosen to measure the momentum distribution of the es. This would have let to a momentum dependent wavefunction ACFst).

From the stit-based thought explo-,

we also anlicipate that the order

in which we combine these states

does not matter and all should yield

For that matter we could have measured the distribution of any dynamical properly and alsociated a wave function with All these would, of course, describe the same state (same preparation of oysteu). This means that there is an object, more fundamental that any of the wave function, that identifies the state. Let's denote this object as 147, where 17 is called a ket, and & inside reminds us that this state is associated with the wavefunction 40,t).

We can necast everything we are about to discuss in terms of these kets with the equivalence II> \\Implies \forms (\text{fist}).

But for now lets just dick to wavefuling

The properties of QM states littled out in postulate 1 are remniscent of a mathematical construct called a vector space which we will review raws.

Vector Spaces: A collection of, objects,

W, 127, 127, 127, ..., IW? (the ket symbol
being a personal choice not to be confused
with the QM state), for which there
exists,

a) A definite rule for combining two objects (sum paddition), denoted (V)+ (W)

b) A definite rule for multiplication by "scalars" a, b, ..., denoted a IV).

is said to form a vector space iff.

1. The result of each of the operations above is an object in the same collection. [closure] Salvy +blw> EV Vector sum

Scalars

Real or complex.

2. Scalar multiplication is (a) distributive in the vectors a(147+1447) = a147+a147

(b) distributive in scalars
(a+b) |V> = a|V>+ bN>

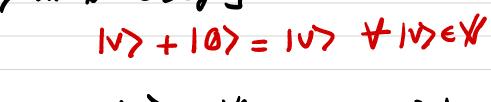
(c) associative a (11/7) = (eb) //>

3. (Vector) addition is

(a) Commutative

$$|V\rangle + |W\rangle = |W\rangle + |V\rangle$$





5. For every 
$$|V\rangle \in V$$
, there coints an inverse  $|-V\rangle \in V$ , such that

This just implies:  

$$1-47 = 10$$
 +  $(-1)14$ 

= (-1) 1/2 .

These arisms define live objects as "vector" in their "Vector space". They imply (show): -> 10) is unique. ie. 4 a 10'> satisfies être same properties as 10) lten |0>= |0'> > 0 | V > = 10 > (see above) → 1-V> = -1V> -> 1-47 is hence, unique additue inverse. Examples: a) cartesian vectors in 3-d space

b) Set of all 2x2 matrices

c) Set of all functions 
$$f(z)$$
,  $0 \le z \le L$ 

such that  $f(z=0) = f(z=L) = 0$ 

Most importantly, we recognize that QM wavefunctions also form a vector space!